Day 7 - Probability I

Introduction

In this section, we focus on understanding the fundamentals of probability. By the end of this activity, you should be familliar with the following terms:

- random process
- sample space
- event
- probability
- probability distribution
- union
- joint
- intersection
- disjoint
- complementary
- independent

Warm-up

Suppose I flip a fair coin 10 times.

- Before I actually flip the coins, how many heads do you *expect* to see?
- I actually flipped them, what's the chance we see exactly 5 heads?

Foundations of probability

Note

A random process is a process for which we can list all the possible outcomes, but none can be predicted with certainty.

Examples:

- Flipping a coin
- Rolling a die
- The # of dead students of blue eyes?
- proportion of bats in out of cars

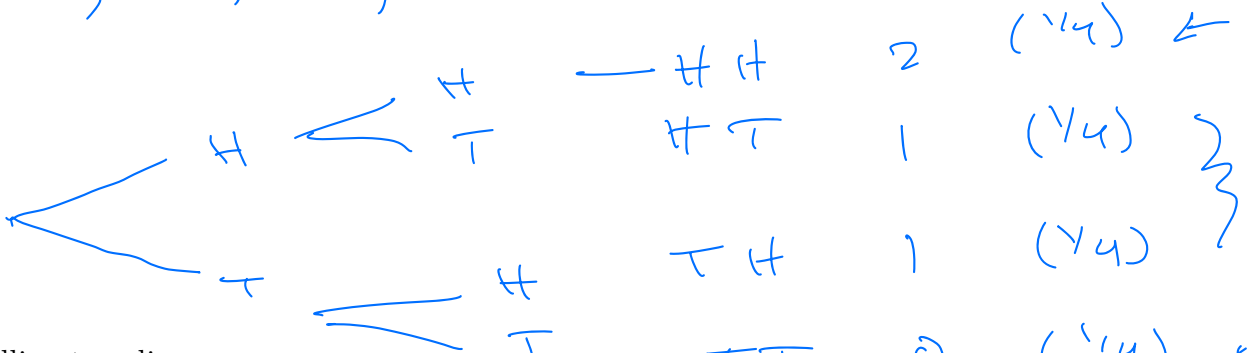
Note

The sample space is the list of all possible outcomes, called **events**, that can result from a random process, often denoted $\mathcal{S}$. Events are typically denoted with capital letters, i.e. A.

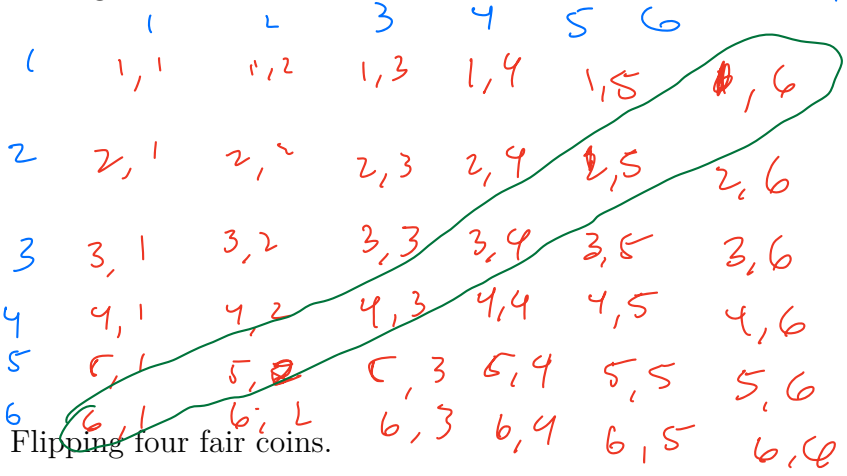
List the sample space for the following random processes:

1. Flipping two fair coins.

$S = \{HH, HT, TH, TT\}$

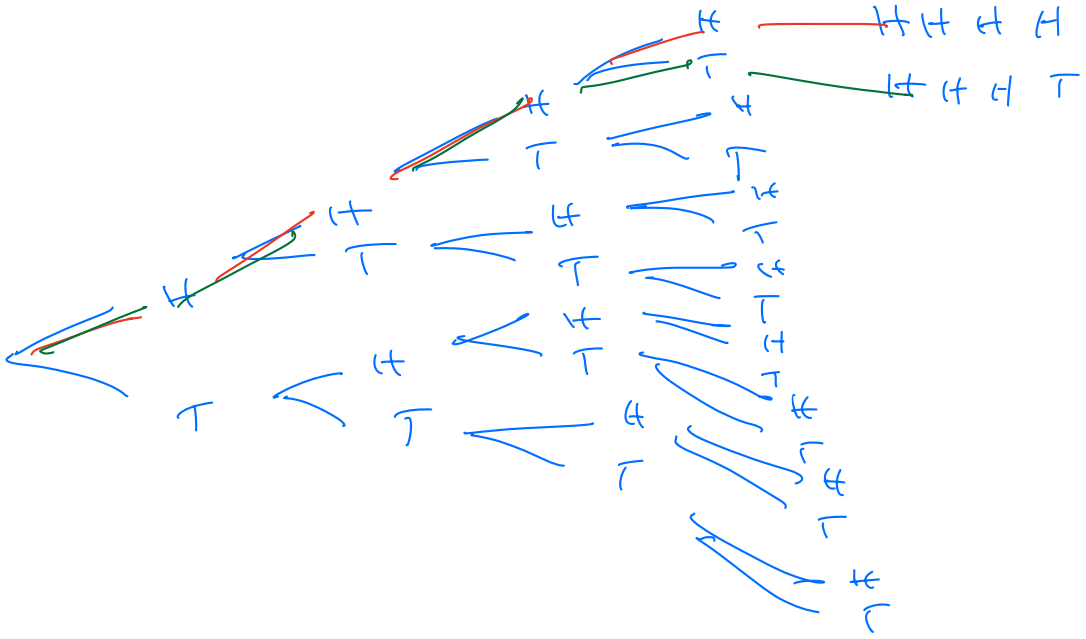


2. Rolling two dice.



$P(\text{sum} = 7) = \frac{6}{36}$

3. Flipping four fair coins.



Note

The probability of an event A is defined as the proportion of times the outcome A would be observed if we could repeat the random process an infinite number of times, denoted P(A).

$$f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}$$

$0 \leq P(A) \leq 1$

A probability distribution describes the probability of observing all possible outcomes of a random process.

Example: A probability distribution from 2010 U.S. Census.

- All probs sum to 1

Size of household	1	2	3	4	5	6	7 or more
Probability	0.267	0.336	0.158	0.137	0.063	0.024	0.015

P(2 or fewer) =

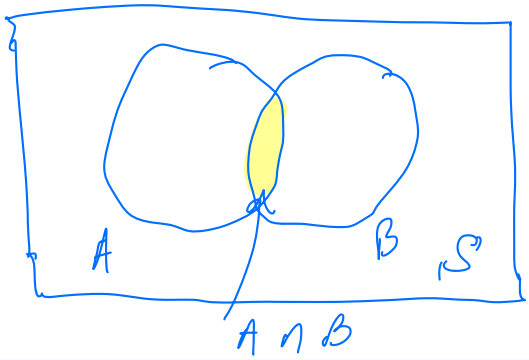
Suppose X denotes the number of heads resulting from flipping two fair coins. Write out the probability distribution for X.

X	0	1	2
$P(X = x)$	.25	.50	.25

Interactions of events

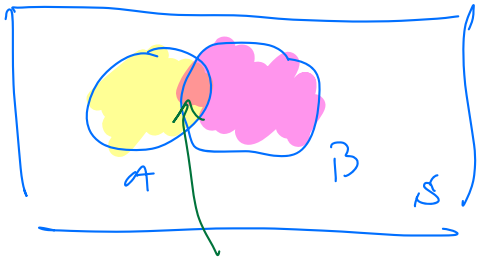
Note

The intersection of A and B, written $A \cap B$ ("A *and* B"), is the probability that both A and B occur. It is also called the joint probability of A and B.



Note

The union of A and B, written $A \cup B$ ("A *or* B"), is the probability that either A occurs, or B occurs, *or both* A and B occur.



Note

The **addition rule** states that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example

A survey was distributed to (fictional) students inquiring about their use of marijuana and whether they share their parents' political beliefs. The results are summarize in the table below.

		Share parents' politics		
	Legalize MJ	No	Yes	Sum
A'	No	11	40	51
A	Yes	36	78	114
	Sum	47	118	165

A: legalize MJ
A': don't legalize
B: share parents political
B': don't

1. What is the probability that a randomly sampled student thinks marijuana (MJ) should be legalized?

$P(A) = \frac{114}{165}$ "Marginal probability"

2. What is the probability that a randomly sampled student shares their parents' political views?

$P(B) = \frac{118}{165}$, marginal probability

3. Calculate the joint probability that a randomly sampled student both supports legalizing marijuana AND shares their parents' political views.

$P(A \cap B) = \frac{78}{165}$, joint

4. What is the probability that a randomly sampled student supports legalizing marijuana OR shares their parents political views (or both)?

$P(A \cup B) = \frac{36 + 78 + 40}{165}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Note

Events A and B are called disjoint, or **mutually exclusive**, if the occurrence of event A precludes the occurrence of event B. For disjoint events,

$$P(A \cap B) = 0$$

Examples:

A: it is day, B: it is night

A: Tails, B: Heads

A: Even roll, B: odd roll

Note

Complentary events are two mutually exclusive events whose probabilities sum to one.

Suppose we roll a fair die and let A be the event of rolling an even number. What is A'?

A': odd number

Note

Two processes are independent if the knowledge of the outcome of one event provides no information about the outcome of the other event.

If events A and B are independent,

$$P(A \cap B) = P(A) \cdot P(B)$$